UNIT-IV

DESIGN FOR FLEXURAL RESISTANCE

Types of flexural failure :-

It depends upon

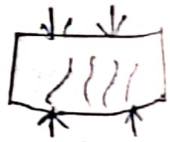
1. % of reinforcement in the section.
2. Bond b/w concrete and steel/tension.
3. Compressive strength of concrete.
4. Ultimate tensile strength of tendon.

Types :-

1. Fracture of steel in tendon
2. Failure of under reinforcement section.
3. Failure of over reinforced section.
4. Other modes of failure

(2)

(3)



Pretensioned → Inadequate transmission length.

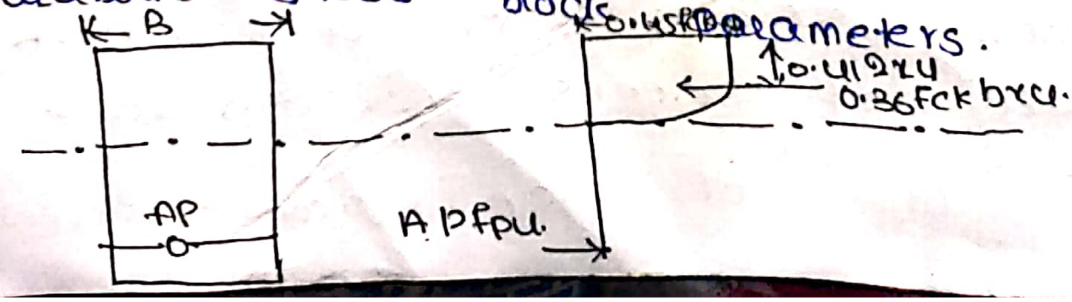
Post tensioned → Anchorage failure

IS code provisions :-

IS:1343-1980 : Appendix-B
Pg:59,60.

Rectangular Section:-

The Indian Standard code method for computing the flexural strength of rectangular sections (or) T-sections in which the neutral axis lies with in the flange, is based on the rectangle and parabolic stress block parameters.



$M = f_{pu} A_p [d - 0.42 x_u]$.

where,

m = moment of resistance of the section.

f_{pu} = ultimate tensile stress in the tendons.

A_p = Area of Pre stressing tendon.

d = effective depth.

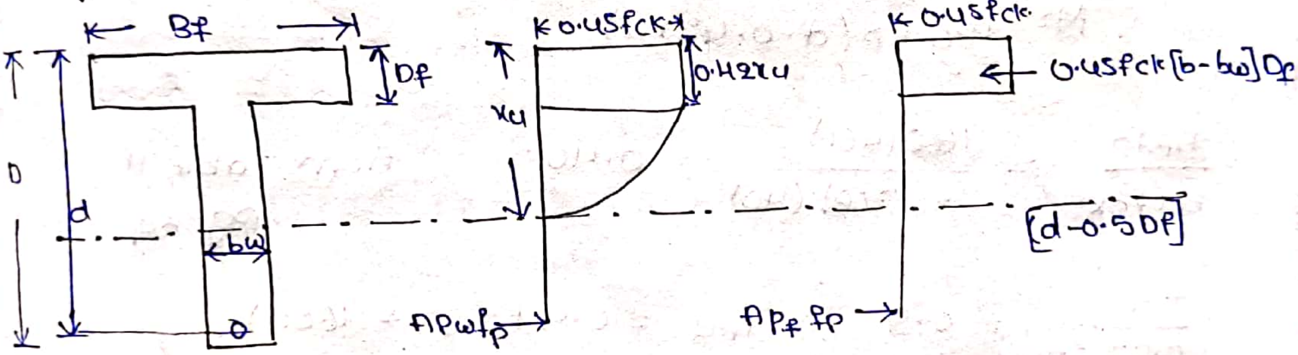
x_u = neutral axis depth.

Based on the value of $\frac{A_p f_p}{bd f_{ck}}$ on.

The value of $\frac{f_{pu}}{f_p}$, x_u are obtained from Table :11 of IS:1343-1980 NXB Pg: 59 & 60.

1/2/2020 $f_{pu} \not> 0.45 f_b$.

T-Section : [moment of resistance of flanged section]



where $x_u > D_f$

$M_U = f_{pu} A_{pw} [d - 0.42 x_u] + 0.45 f_{ck} [b - b_w] D_f [d - 0.5 D_f]$

$A_{pf} = 0.45 f_{ck} [b - b_w] \left[\frac{D_f}{f_p}\right]$

then $A_{pw} = A_p - A_{pf}$

effective reinforcement ratio $\frac{A_{pw} f_p}{b d f_{ck}}$ (IS:1343-1980) * $b_r = b_w$.

A_{pw} = Area of Prestressing steel for Web.

A_{pf} = Area of Prestressing steel for flange.

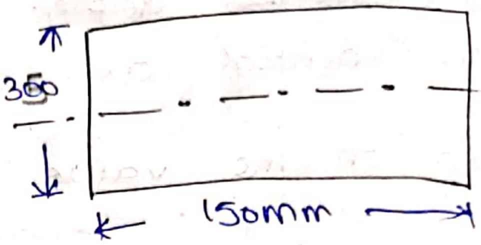
D_f = thickness of flange.

b_w = thickness of web.

The corresponding values of $\frac{f_{pu}}{0.87 f_b}$ & $\frac{x_u}{d}$ are obtained from table 11 pg: 59.

of 50mm. If $f_{ck} = 40\,N/mm^2$ and $f_p = 1600\,N/mm^2$ and the area of prestressing steel $(A_p) = 461\,mm^2$. Calculate the ultimate flexural strength of the section using I.S. code method.

Sol: Given data,



Wide (b) = 150mm

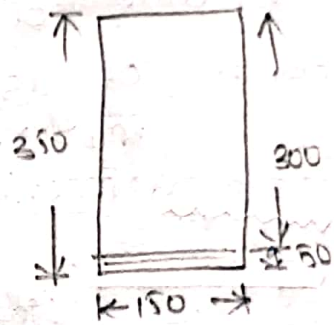
Deep (D) = 350mm

effective cover = 50mm

$f_{ck} = 40\,N/mm^2$

$f_p = 1600\,N/mm^2$

$A_p = 461\,mm^2$



effective depth (d) = 350 − 50 = 300mm

$M = f_{pu} A_p (d - 0.42 x_u)$

$$\frac{A_p f_p}{b d f_{ck}} = \frac{1600(461)}{150(300)(40)} = 0.40$$

From Table 11 Pg: 59

$$\frac{f_{pu}}{0.87 f_p} = 0.9 \Rightarrow f_{pu} = 0.9(0.87 * 1600)$$

$f_{pu} = 1252.8\,N/mm^2$

$$\frac{x_u}{d} = 0.783 \Rightarrow 0.783(300) = 234.9\,mm$$

$M = f_{pu} A_p (d - 0.42 x_u)$

$= 1252.8(461)[300 - (0.42 * 234.9)]$

$= 577540 * [201.3]$

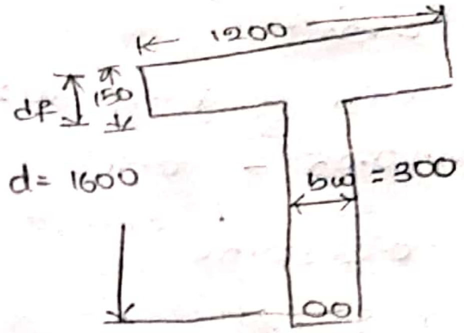
$\boxed{M = 116 \times 10^{6}\ kN\text{-}m}$

7/2/2020.

2) A pretensioned T-section has a flange of 1200mm wide & 150mm thick. The width & depth of the Rib 300mm and 150mm resp. The high tensile steel as an area of $4700mm^2$ and is located at an effective depth of 1600mm, If the characteristic strength of the cube concrete & tensile strength of steel is

, 40 N/mm^2 & 1600 N/mm^2 resp. Calculate the flexural strength of T-section.

Given data,



b_f = 1200mm

d_f = 150mm.

b_w = 300mm

bdo = 150mm.

Area $(A_p) = 4700mm^2$

effective depth (d) = 1600mm.

f_{ck} = 40N/mm^2

Tensile strength (f_p) = 1600N/mm^2.

$$\text{Effective reinforcement ratio} = \frac{A_{pw} f_p}{b d f_{ck}}$$

$$= \frac{}{(4700)(300)(1600)}$$

$A_{pw} = A_p - A_{pf}$

then $A_{pf} = 0.45 f_{ck} [b_f - b_w] \left[\frac{D_f}{f_p}\right]$

$$= 0.45(40)[1200-300]\left[\frac{150}{1600}\right]$$

$= 1518.75mm^2$.

$A_{pw} = 4700 - 1518.75$

$= 3181.25mm^2$

$\Rightarrow \dfrac{3181.25(1600)}{300(1600)(40)} = 0.26$.

0.25 – 1   Interpolation = 1.0.

0.30 – 1

$\dfrac{f_{pu}}{0.87 f_p} = 1$

$\Rightarrow f_{pu} = (1)(0.87)(1600) = 1392$ N/mm^2

$\dfrac{x_u}{d} = 0.265$.

0.25 – 0.542 → (1)

0.26 – x → (2)

0.30 – 0.655 → (3)

(2) − (1) $\qquad\qquad\qquad\qquad$ $1.1*10$ ... 0.027

$\cancel{0.25x}$ −

(2)(1) 0.01 = $x - (0.542)$ $\qquad\qquad$ $x = \dfrac{1.1*10^{-3}+0.027}{0.05}$

(3)-(1) 0.05 = 0.113 $\qquad\qquad$ $\boxed{x = 0.56}$

0.26 = 0.56

$\dfrac{x_u}{d} = 0.56$

$x_u = 0.56(1600) = 896\,mm.$

$D_f = 150\,mm.$

Where

$x_u > D_f.$

$M_U = f_{pu}A_{pw}[d - 0.42x_u] + 0.45 f_{ck}[b - b_w]D_f[d - 0.5D_f]$

$= (1392)(318.25)[1600 - 0.42(896)] + 0.45(40)[1200 - 300]$

$150[1600 - 0.5(150)]$

$= 442830(1223.68) + 16200(228750)$

$M_U = 9124*10^{6}\ kN\text{-}m$

$\boxed{M_U = 9124.5\ kN\text{-}m}$

7/9/2020.

3) A post tensioned prestressed concrete T-beam having a flange width of 1200mm and thickness of flange 200mm, thickness of web deep be 300mm is prestressed by 2000mm² of high tensile steel located at an effective depth of 1600mm. If $f_{ck} = 40\,N/mm^2$, $f_p = 1600\,N/mm^2$, estimate the ultimate flexural strength of the unbonded T-section - Assuming span to depth ratio as 20 and $f_{pe} = 1000\,N/mm^2$

Sol: Given data.

$b_f = 1200\,mm$

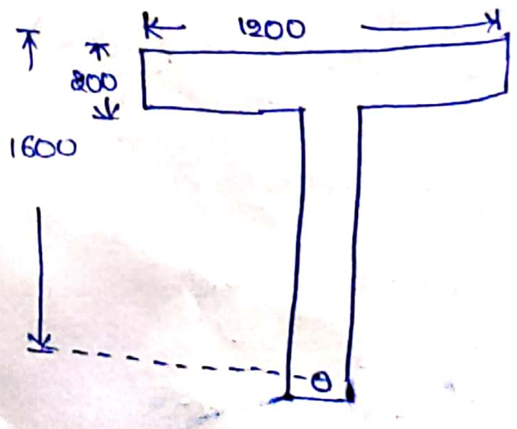
$d_f = 200\,mm.$

$t_w = 300\,mm$

$A_p = 2000\,mm^2$

effective depth(d) = 1600mm.

$f_{ck} = 40\,N/mm^2$



$f_{pe} = 1000\ N/mm^2$

$\frac{l}{d} = 20$.

Assumming the Neutral axis to fall within the flange. We have the ratio of

$$= \frac{A_p f_{pe}}{bd\ f_{ck}}$$

$$= \frac{2000(1000)}{1200 * 1600 * 40}$$

$$= 0.026$$

0.025 – 1.34

0.026 – x

0.05 – 1.32

consider 1.34.

$$\Rightarrow \frac{f_{pu}}{f_{pe}} = 1.34$$

$$f_{pu} = 1.34[1000] = \underline{1340}\ N/mm^2.$$

$$\Rightarrow \frac{x_u}{d} = 0.10$$

$$x_u = 0.10(1600)$$

$$\boxed{x_u = 160mm}$$

$x_u < d_f$.

consider

$$M = f_{pu} A_p [d - 0.42 x_u]$$

$$= 1340(2000)[1600 - 0.42(160)]$$

$$= 2680000(1532.8)$$

$$\boxed{M = 4107\ kN\text{-}m}$$

4) A Post-tensioned bride girder with unbonded tendons is of box section of overall dimension 1200mm wide x 1800mm deep with wall thickness of 150mm and high tensile steel has an area of 4000mm² and is located at an effective depth of 1600mm. The effective Prestress in steel after all losses is 1000N/mm² (f_{pe}), effective span of girder is 24m. If $f_{ck} = 40N/mm^2$ and $f_{pu} = 1600\ N/mm^2$. Estimate the ultimate flexural strength of section. Assume $b_w = 300mm$.

Sol: Given data:

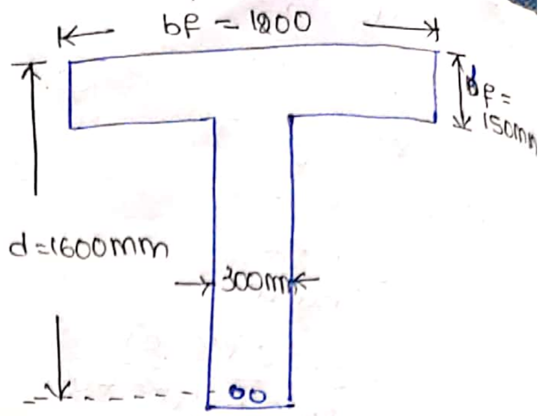
$d_f = 150mm$

$A_p = 4000mm^2$

effective depth $(d) = 1600mm$

$f_{pe} = 1000 N/mm^2$

span length = 24m



$b_f = 1200mm$; $D_f = 150mm$

$d_f = 1800mm$.

$b_w = 300mm$.

$f_{pu} = 1600 N/mm^2$

$f_{ck} = 40 N/mm^2$.

Effective reinforcement ratio $= \frac{A_{pw} f_p}{b d f_{ck}}$.

$A_{pw} = A_p - A_{pf}$

$A_{pf} = 0.45 f_{ck} [b_f - b_w] \left[\frac{D_f}{f_p}\right]$

$= 0.45 (40) [1200 - 300] \left[\frac{150}{1600}\right]$

$= 1518.75 mm^2$

$A_{pw} = A_p - A_{pf}$

$= 4000 - 1518.75$

$= 2481.25 mm^2$

$\frac{2481.25 (1800)}{300 (1600)(40)} = 0.129$

$\frac{l}{d} = \frac{24 \times 10^3}{1600} = 15$ Consider $\frac{l}{d} = 10$.

0.10 — 1.45 → ①

0.129 — x → ②

0.15 — 1.36 → ③

(2) − (1) ⇒ 0.029 = x − 1.45

(3) − (1) → 0.05 = −0.09

$-0.0018_{\nearrow} = 0.05x - 0.0725$

-2.61×10^{-3}

$D_f = 150mm$

$\boxed{x = 1.38}$

$\frac{f_{pu}}{f_{pe}} = 1.38$

$f_{pu} = 1.038(1000)$

$= \underline{1380} \; N/mm^2$

$\frac{x_u}{d} = 0.129.$

0.10 — 0.36
0.129 — x
0.15 — 0.52.

$0.52 + \frac{[0.52 - 0.36]}{0.15 - 0.10} * (0.129 - 0.15)$ 0.021

$= 0.45$

$\frac{x_u}{d} = 0.45$

$x_u = 0.45(1600)$

$= 720\ mm$

$x_u > D_f.$

$M_u = f_p A_{pw}[d - 0.42x_u] + 0.45 f_{ck}[b - b_w]D_f [d - 0.5D_f]$

$= (1380)(2481.25)[1600 - 0.42(720)] +$
$0.45(40)[1200 - 300]150[1600 - 0.5(150)]$

$= 4443.4 \times 10^6 + 3705.75 \times 10^6$

$\boxed{M_u = 8148\ kN\text{-}m}$

10/2/2020

Main reasons for controlling deflection:

Large Deflections:

→ Under dynamic effect and unk

→ Under variable loading.

Excessive Deflections:

→ Damage to finishers

→ Partitions

→ Associated structures.

** Factors influencing deflection :

1) Imposed load and self weight.
2) magnitude of Prestressing force.
3) span of member.
4) Cable profile.
5) young's modolous of concrete.
6) moment of Inertia (or) 2nd moment of area of cross-section.
7) shrinkage, creep and releaxation of stress
8) Fixidity conditions.

Calculation of deflections :

These are two types.

1. Post cracking

→ It is similar R.C.C
R.C.C is " to P.S.C.
→ moment curvature

2. Pre cracking.

→ mom whole section is conside moment using mohr's 2nd theorem.

Deflection calculation : 2nd moment area of c/s sectio (or) 2nd moment Inertia.

Mohr's first law:

$$\text{slope} = \frac{\text{Area of BMD}}{\text{Flexural rigidity}}$$

$$\theta = \frac{A}{EI}$$

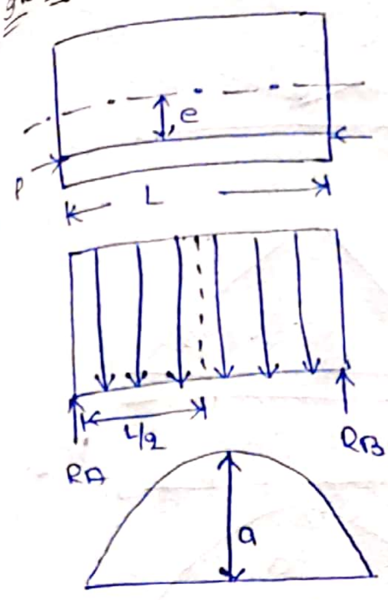
mohr's second law:

$$a = \frac{\text{moment of area of BMD}}{\text{flexural rigidity}}$$

$$= \frac{A\bar{x}}{EI}.$$

Effect of tendon profile on deflection :-

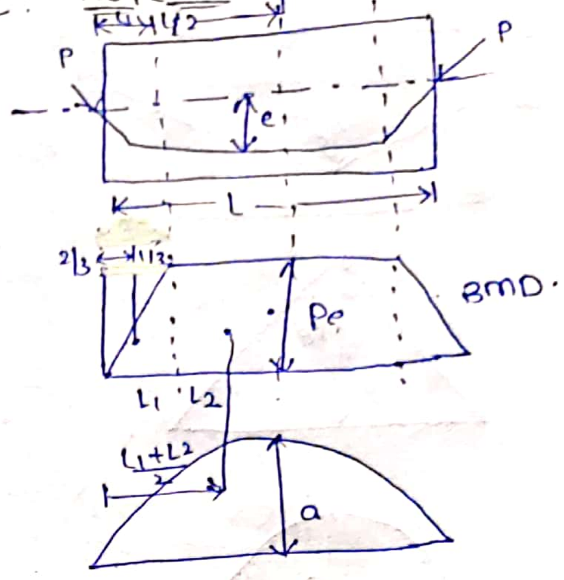
Straight tendon :



$$a = \frac{PeL^2}{8EI} \text{ (upward).}$$

11/9/2020

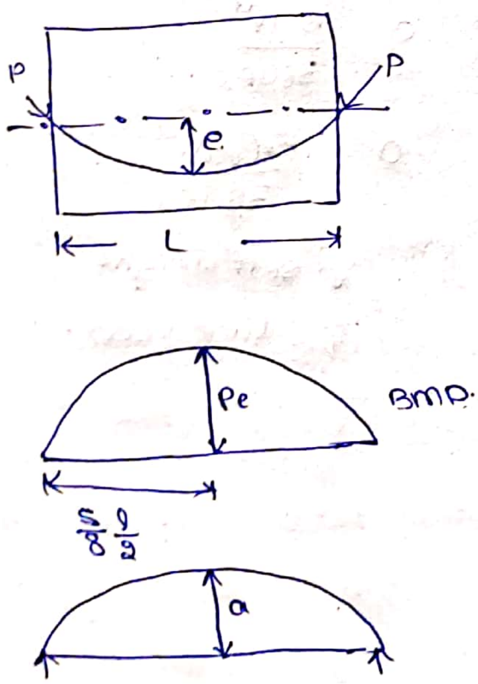
Trapezoidal Tendon:



$$a = \frac{-Pe}{6EI}\left[2l_1^2 + 6l_1 l_2 + 3l_2^2\right]$$

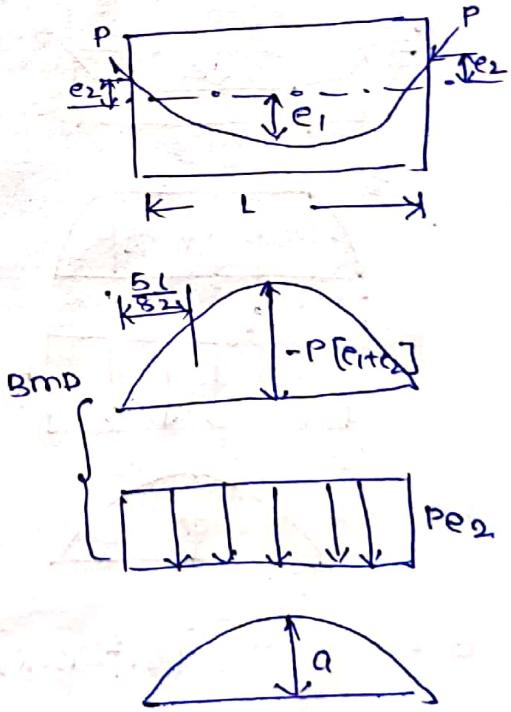
3. Parabolic tendon

1) center



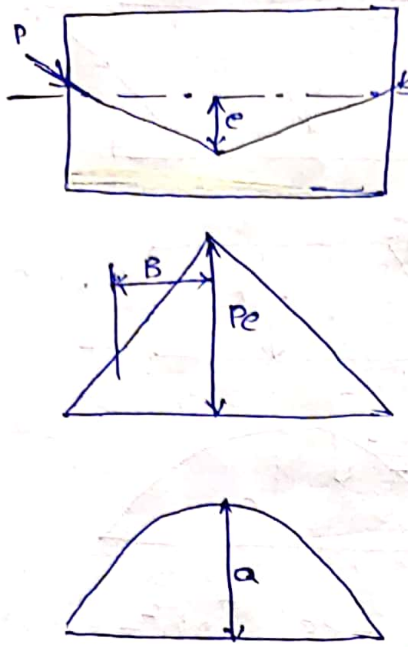
$$a = \frac{-5Pel^2}{48EI}.$$

2) eccentric



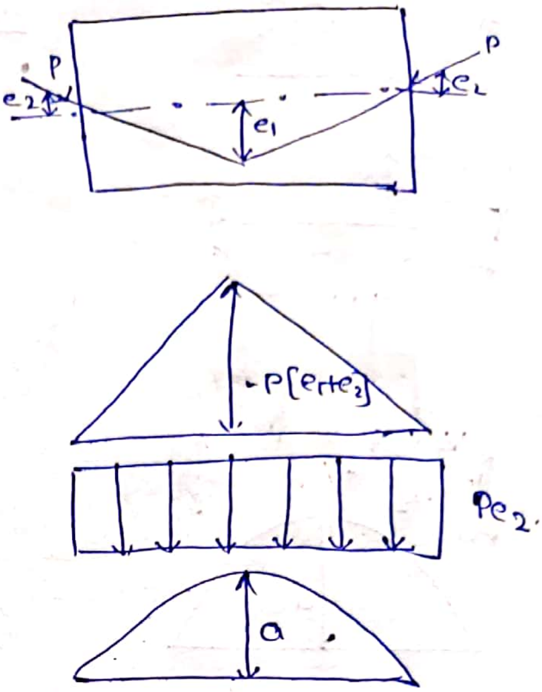
$$a = -\frac{Pl^2}{48EI}\left[-5e_1 + e_2\right]$$

Triangular Tendon:



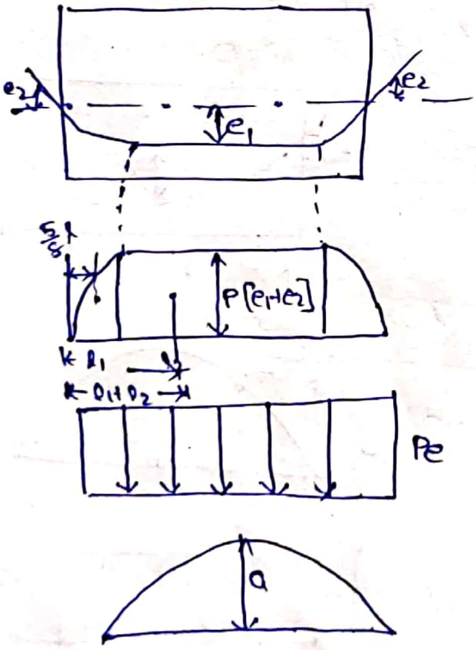
$$a = \frac{Pel^2}{12EI}$$

Slopping tendon



$$a = \frac{Pl^2}{24EI}\left[-2e_1+e_2\right].$$

Parabolic Straight tendon.



$$a = \frac{-P(e_1+e_2)}{12EI}\left[5l_1^2+12l_1l_2+6l_2^2\right]+\frac{Pe_2l^2}{8EI}.$$

Deflection due to selfweight & imposed load

$$a = \frac{5wl^4}{384EI}$$

$$a = \frac{5(g+q)l^4}{384EI}.$$

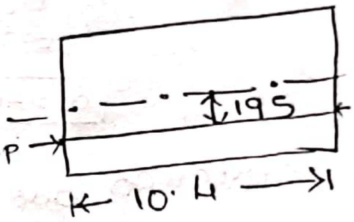
g = self weight

q = live load.

1. The deck of a Prestressed concrete colverto r made up a slab 500mm thick. The slab is spanning over 10.4m and supports a total UDL compressing the dead & live loads of 23.5 kN/m. The modolous of elasticity of concrete (Ec) = 38kN/mm². The concrete slab is prestressed by straight cables each containing 12 high tensile wires of 7mm dia. is stressed to 1200 N/mm² at a constant eccentricity of 195mm. The cables are spaced at 328mm intervals in the transverse direction. Estimate the instantaneous deflections of the slab at the centre of the span under prestressed & imposed loads.

1/3/2020

Sol: Given data,



Thickness of slab = 500 mm

stress = 1200 N/mm²

Span length (l) = 10.4m.

Load (dead + live) (g+q) = 23.5 kN/m.

Ec = 38 kN/mm²

No. of wires = 12 wires - 7mm ∅.

eccentricity (e) = 195mm.

Spacing of cable in transverse direction = 328 mm.

Assume width (b) = 1000 mm.

Force in each cable (F) = stress × Area

Area (A) = $n\frac{\pi}{4}(d)^2$

$= 12\ \frac{\pi}{4}(7)^2 = 461.81$

F = 461.81 × 12

$\boxed{P = 554.17\ kN}$

Hence the prestressing force per meter width of slab is compound as

$$P = \frac{1000 \times 554.17}{328}$$

$$\boxed{P = 1689.54 \text{ KN}}$$

Deflection due to prestressing -force (straight cable)

$$a = -\frac{Pel^2}{8EI}.$$

$$I = \frac{bd^3}{12} = \frac{(1000)(500)^3}{12} = 10416.66 \times 10^6 \text{ mm}^4.$$

$$a = -\frac{(1689.54)(195)(10.4 \times 10^3)^2}{8 \times 38 \times 10416.66 \times 10^6}$$

$$= \cancel{9.00} \text{ mm}$$

$$\boxed{a = -11.25\text{mm}} \uparrow \text{(-ve)}$$

Deflection due to selfweight and deadload

$$a = \frac{5(q+g)l^4}{384EI}.$$

$$= \frac{5(23.5 \times 10^3)(10.4 \times 10^3)^4}{384 \times 38 \times 10416.66 \times 10^6}$$

$$\boxed{a = 9.043\text{mm}} \text{ (due to loads +ve)}.$$

Resultant deflection $= 9.043 - 11.25$

$$\boxed{a = -2.20 \text{ mm}} \uparrow$$

2) A PSC beam of rectangular beam 120mm wide & 300mm deep span over 6m the beam is prestressed by a straight cable. carrying an effective force of 200kN at an eccentricity of 50mm. $E_c = 38 \text{ kN/mm}^2$ compute the deflection at centre of span for following cases.

i) Deflection under prestress + selfweight.

ii) Find the magnitude of UDL live load which will nullify the deflection due to prestress & selfweight.

Sol: Given data,

$b = 120mm$

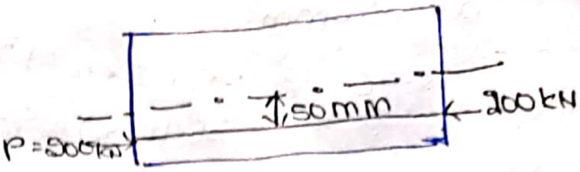
$d = 300mm$.

length $(l) = 6m$.

force $(P) = 200kN$.

$e = 50mm$

$E_c = 38 kN/mm^2$



$$\text{stress} = \frac{\text{force}}{\text{Area}} = \frac{200*10^3}{120*300}.$$

$$\text{stress}(F) = 5N/mm^2.$$

Deflection due to Prestressing force [st. cable]

i)

$$a = \frac{-Pel^2}{8EI}$$

$$I = \frac{bd^3}{12} = \frac{(120)(300)^3}{12} = 270*10^6 mm^4$$

$$a = -\frac{(200)(50)(6*10^3)^2}{8*38*270*10^6}.$$

$$\boxed{a = -4.38mm}$$

Deflection due to selfweight.

$$a = \frac{5wl^4}{384EI}.$$

$$w = 0.12*0.3*24 = 0.86 \ kN/m.$$

$$a = \frac{5(0.86*10^{-3})(6*10^3)^4}{384(38)(270*10^6)}$$

$$\boxed{a = 1.42mm}$$

Resultant deflection $= 1.42 - 4.38$

3/12/2020

$$\boxed{a = -2.96mm}$$

ii) Deflection due to live load.

$$a = \frac{5wl^4}{384EI} \qquad w = 9$$

$$a = \frac{59l^4}{384EI}$$

'a' can be taken as +ve; and resultant deflection

$a = 2.96mm$

$$2.96 = \frac{5(q)(6*10^3)^4}{384(38)(270*10^6)}$$

$1m = 1000mm$

$\frac{1}{1000} m = 1mm$

$kN/m \leftarrow N/m$

$\boxed{q = 1.8 N/mm}$

$\boxed{q = 1.8 kN/m.}$

3) A rectangular beam of c/s section 150mm & 300mm deep is simply supported over a span of 8m & is prestressed by means of a symmetric parabolic cable at a distance of 75mm from the bottom of the beam, at mid span & 125mm from the top of the beam at the support sections. If the force in the cable is 350kN and $E_c = 38 kN/mm^2$

Calculate :(a) The deflection at mid span when the beam is supporting its own weight.

(b) The concentrated load which must be applied at midspan to restore it to the level of support

Sol: Given data, $(e_2 = 150 - 125 = 25)$

$E_c = 38 kN/mm^2$

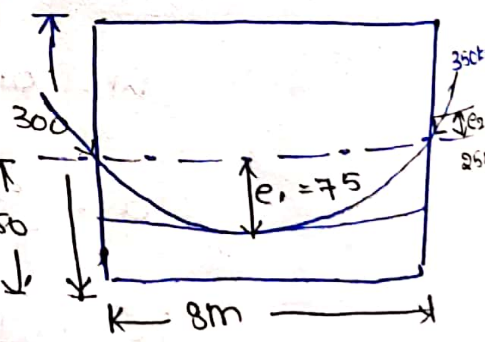
$P = 350 kN$

$c/s = 150 * 300mm$

$L = 8m$

$e_1 = 75mm$

$e_2 = 25mm.$



a) Deflection due to prestressing force [Parabolic eccentricity tendons]

$$a = \frac{+Pl^2}{48EI}[-5e_1 + e_2]$$

$$I = \frac{bd^3}{12} = \frac{150*300^3}{12} \Rightarrow \boxed{I = 337.5*10^6 \; mm^4}$$

$$a = + \frac{350(8*10^3)^2}{48(38)(337.5*10^6)} [-5(75+1)+25]$$

$= 0.036(-350)$

$$\boxed{a = -12.6 mm} \uparrow$$

deflection due to self weight

$$a = \frac{5ql^4}{384EI}$$

$q = 24*0.15*0.3 = 1.08$ KN/m.

$q = 1.08$ N/mm.

def

$$a = \frac{5(1.08)(8*10^3)^4}{384(38*10^3)(337.5*10^6)}$$

$$\boxed{a = 4.49 mm} \downarrow \text{(downward)}$$

Prestress at self weight $= 4.49 - 12.6$

$$\boxed{a = 8.11 mm}$$

(b) concentrated load

$$a = \frac{Wl^3}{48EI} \qquad W = q$$

$$q = \frac{48EIa}{l^3}$$

$$= \frac{48*38*337.5*10^6*8.11}{(8*10^3)^3}$$

$$\boxed{q = 9.74 kN}$$

1/2/2020

LONG TERM DEFLECTIONS:

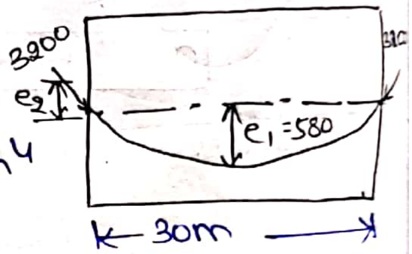
4) A post tensioned roof girder spaning over 30m has an unsymmetrical I-section with a 2nd moment area of 72,490 * 10^6 mm^4 (I) & an overall depth of 1300mm. The effective eccentricity of the group of parabolic cables at a centre of span is 580mm towards the

soffit and 170mm towards the top of the beam at supports. The cables carries an intial prestressing force of 3200 kN. The self weight of the girder 10.8kN/m and the live load of the girdel is 9kN/m. The modolous of elasicity of concrete is 34 kN/mm². If the creep coefficient is 1.6 and the total loss of prestress is 15%. Estimate the deflections of the following stages & compale them with the permissible values according to IS code: 1343 limits.

a) Instaneous deflection due to prestress + self weight

b) Resultant maximum long term deflection allowing for loss of prestress and creep of concrete.

Sol: Given data,



span(l) = 30m.

2nd moment area (I) = 78490×10^6 mm⁴

overall depth (d) = 1300mm

$e_1 = 580$mm

$e_2 = 170$mm

Force (P) = 3200 kN.

Self weight of girder (g) = 10.8 kN/m.

live load (q) = 9kN/m.

$E_c = 34$ kN/mm².

creep coefficient(ϕ) = 1.6.

Total loss of prestress is 15%.

a) Prestress + self weigth.

$$a = \frac{Pl^2}{48EI}[-5e_1 + e_2]$$

$$= \frac{3200(30 \times 10^3)^2}{48(34)(78490 \times 10^6)}[-5(580)+170]$$

$$\boxed{a = -66.45 \text{ mm}} \uparrow \text{(upward)}$$

Deflection due to self weight.

$$a = \frac{5ql^4}{384EI}$$

$$= \frac{5\left(\frac{10.8}{9}\right)(30*10^3)^4}{384(38)(72490*10^6)}$$

$$\boxed{a = 46.21\ mm}\downarrow$$

Prestress at self weight. = 46.21 − 66.45

$$\boxed{a = -20.23\ mm}\updownarrow$$

$2|2020$

Deflection due to live load.

$$a = \frac{5ql^4}{384EI}.$$

$$\frac{20.23}{a} = \frac{5\left(\frac{9}{9}\right)(30*10^3)^4}{384\ (38)(7249*10^6)}$$

$$a = \frac{5ql^4}{384EI}$$

$$a = \frac{5(9)(30*10^3)^4}{384*38*7249*10^6}.$$

$$\boxed{a = 38.51\ mm}$$

Long term deflection due to creep.

ai = self weight
af = final

$$af = [1+\phi]ai$$

$$af = [1+1.6][46.21]$$

$$af = 120.14.$$

15% loss of prestress

100 − 15 = 85%

$$\frac{85}{100} = 0.85$$

$$= 0.85*P$$

$$= 0.85 * -66.45$$

$$= -56.48$$

Total resultant long term deflection =

38.51 + 120.14 − 56.48

= 102.16 mm

From IS 1343 : Pg: 32 cl: 19.3.1 (or) 20.3.1 above value should not be exceed $\frac{span}{250} = \frac{30*10^3}{250}$

= 120 mm

102.16 < 120 mm.

Hence ok.

17/01/2020

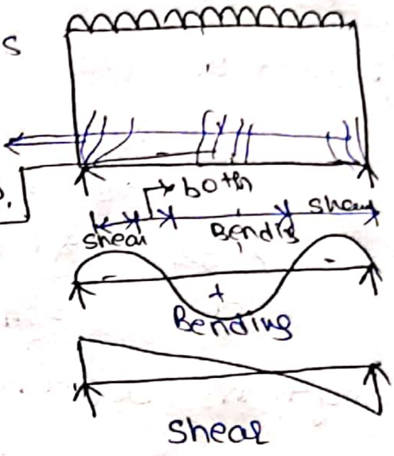
UNIT-5

DESIGN FOR SHEAR & TORSION

Shear and principle stresses:

The shear distribution in an uncracked structural member for which the deformation is assumed to be linear is the function of shear force and the properties of the cross-section of the member.

The shear stress at a point is expressed as



$$\tau_v = \frac{VA\bar{y}}{Ib} \quad \text{(or)}$$

$$\tau_v = \frac{3}{2}\frac{V_u}{bd}.$$

Where;

τ_v = Shearing stress due to transverse load.

V = shear force

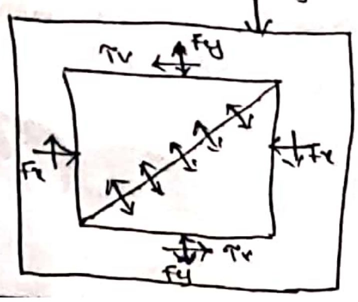
I = moment of Inertia.

b = breadth of member at given point.

In a Psc member the shear stress is generally accompained by a direct stress in the axial direction of the member.

If transverse, vertical prestressing is adopted the compressive stress in the direction perpendicular to the axis of the member will be present in addition to the axial pre-stress. ($V_u - P\sin\theta$)

The most general case of ~~teA~~ an element is subjected to a two-dimensional stress diagram shown in the figure.



Prestress in psc member.

The maximum and minimum principle stress developed are given by f_{max}, f_{min}.

$$f_{max} = \frac{f_x+f_y}{2} + \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

$$f_{min} = \frac{f_x+f_y}{2} - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

where;

f_x & f_y are direct stresses and τ_v is shear stress acting at a point.

In the PSC member the direct stresses f_x & f_y are compressive the magnitude of the principle tensile stresses is considerably reduced and in even some cases even eliminated. So that under working loads both major & minor principle stresses are compressive their by eliminating the disc of diagonal tensional cracks.

In general there are 3 ways of improving the shear resistance of structural ~~memb~~ concrete member by prestressing technique.

1. Horizontal (or) axial prestressing.
2. Prestressing by inclined (or) sloping.
3. Vertical (or) transverse Prestressing.

8/2/2020

1. A PSC beam of span 10m of rectangular section 120mm wide * 300mm deep is axially prestressed by a cable carrying an effective force of 180 KN. The beam supports a total UDL of 5KN/m which includes the selfweight of the member. compare the magnitude of the principle tension developed in the beam with and without axial prestressing.

Sol:

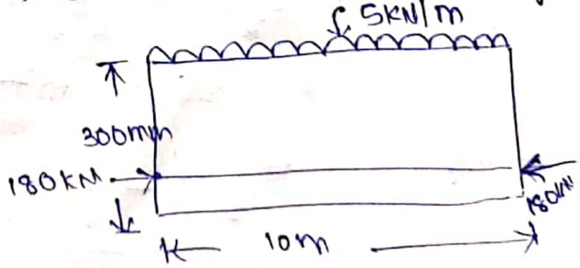
Span (l) = 10m.

Wide (b) = 120mm

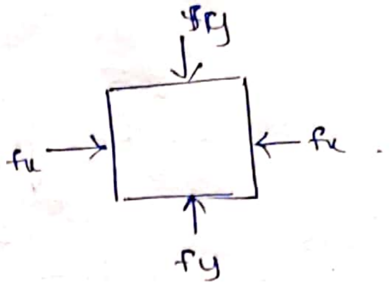
deep (d) = 300mm.

force (F) = 180 KN/m

UDL = 5KN/m.



a) **Without axial Prestressing:**



$f_x = f_y = 0.$

$\tau_v = \frac{3}{8}\frac{V_u}{bd}.$

$V_u = \frac{wl}{2} = \frac{5(10)}{2} = 25\,kN.$

$\tau_v = \frac{3}{2}\frac{(25 \times 10^3)}{120 \times 300}$

$\boxed{\tau_v : 1.04 \ N/mm^2}$

$f_{max} / f_{min} = \frac{f_x + f_y}{2} \pm \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + \tau_v^2}$

Here; $f_x = f_y = 0.$

f_{max} (or) f_{min} $= \pm 1.04 \ N/mm^2$

b) **With axial Prestressing**

$f_x = \frac{\text{Stress} \times \text{Area}}{\cancel{\text{Area}}}$ & $f_y = 0.$

$\frac{P}{A} = \text{stress}$

$\frac{180 \times 10^3}{36 \times 10^3} = \text{stress}$

$\boxed{f_x = 5\,N/mm^2}$

Principle stress

$f_{max}, f_{min};$

$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + \tau_v^2}$

$= \frac{5+0}{2} + \sqrt{\left(\frac{5-0}{2}\right)^2 + (1.041)^2}$

$= 5.2 \ N/mm^2$

$f_{min} = \frac{f_x + f_y}{2} - \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + \tau_v^2}$

$= \frac{5+0}{2} - \sqrt{\left(\frac{5-0}{2}\right)^2 + (1.041)^2}$

$f_{min} = -0.204 \ N/mm^2$

Comparing magnitude of principal tension.

$$\frac{T_v - f_{min}}{T_v} \times 100$$

$$= \frac{1.041 - 0.2}{1.041} \times 100$$

$= 80.78\%$.

2) From the above problem instead of axial prestress-acured cable having an eccentricity 100mm at the centre of the span & redusing to zero at supports is used. The effective force in the cable being 180kN Estimate the percentage redusion in the principal tension in comparsion with the case of axial Prestressing.

Sol: Given data,

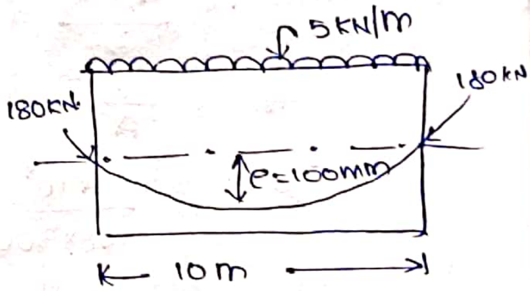
Eccentricity (e) = 100mm.

force (F) = 180 kN.

l = 10m.

wide(b) = 120mm

deep(d) = 300mm.



UDL = 5kN/m.

$V_u - P\sin\theta$.

$\sin\theta$ will be negelible 'θ'

$V_u - P\theta$.

$$\theta = \frac{4e}{L}$$

$$= \frac{4 \times 100}{10 \times 10^3} = 0.04 \text{ rad}.$$

The vertical components of the prestressing force

$= P\sin\theta$.

For smaller values of θ, $\sin\theta$, similar to 'θ' = $P\theta$.

$= 180 \times 10^3 \times 0.04$

$P\theta$ = 7.2 kN

Net sheal at suppoits $V_u - P\theta$.

$$V_u = \frac{wl}{2}$$

$$= \frac{5(10)}{2} = 25\,kN.$$

$$V_u - P\theta = 25 - 7.2$$

$$\boxed{V_u = 17.8\,kN}$$

maximum sheal stress 'τ_v'

$$\tau_v = \frac{3}{2}\frac{V_u}{bd}.$$

$$= \frac{3}{2}\,\frac{17.8 * 10^3}{120 * 300}$$

$$\tau_v = 0.741\ N/mm^2$$

The maximum & minimum principal stress.

$$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + (\tau_v)^2}$$

$$f_x = \frac{P}{A}.$$

$$= \frac{180 * 10^3}{120 * 300}$$

$$= 5\ N/mm^2$$

$$f_{max} = \frac{5+0}{2} + \sqrt{\left(\frac{5-0}{2}\right)^2 + (0.741)^2}$$

$$= 5.10\ N/mm^2\ \text{(compression)}$$

$$f_{min} = \frac{5+0}{2} - \sqrt{\left(\frac{5-0}{2}\right)^2 + (0.741)^2}$$

$$= -\underline{0.107}\ N/mm^2\ \text{(tension)}$$

Compausion both fmin

$$\frac{0.207 - 0.107}{0.207} * 100$$

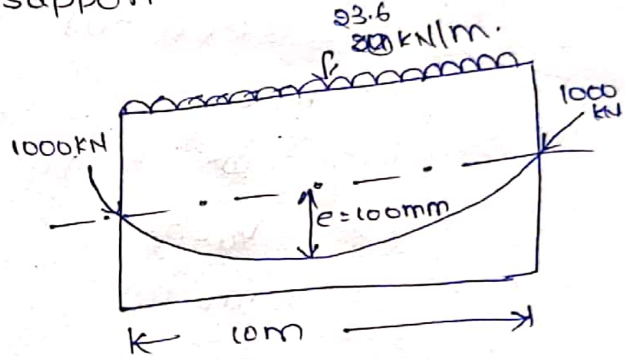
$$= 48\%$$

20/2/2020.

3. A concrete beam rectangular c/s section has a width of 250mm & depth of 600mm the wheel is prestressed by a parabolic cable carrying an effective force of 1000KN. The cable is concentric at supports and has maximum eccentricity 100mm at the centre of the span. The beam spans over 10m & supports a UDL liveload 20 KN/m (a) Assuming the density of concrete is 24KN/m³. estimate the maximum principal stresses developed in a section of beam at a distance of 300mm from the supports. (b) The prestressing force required to nullify the shear-force due to the dead & live loads at the support section.

Sol:

Given data



$P = 1000KN$

$w = 20 KN/m$

$e = 100mm.$

$\rho = 24KN/m^3.$

$c/s = 250 * 600mm.$

$l = 10m.$

The selfweight of the beam = Area * density of concrete.

$= 0.25 \times 0.6 \times 24$

$= 3.6 KN/m.$

Total load on beam (w) = both dead + live load

$= 3.6 + 20$

$= 23.6 KN/m.$

Shear-force at support section.

$$V_u = \frac{wl}{2}$$

$$= \frac{23.6(10)}{2}$$

$$\boxed{V_u = 118 KN}$$

Shear force at a section of 300mm from supports

$V_{at\,300} = V_u - dw$

$= 118 - (0.3)(23.6)$

$V_{at\,300} = 110.92$ KN

$\theta = \frac{4e}{l} = \frac{4(100)}{10 \times 10^3}$

$\theta = 0.04$ rad

Vertical Component of the Prestressing force

$P\sin\theta \simeq P\theta$

$= 1000(0.04)$

$= 40$ KN.

Net shear force at 300mm from the support =

$= V_u - P\theta$

$= 110.92 - 40$

$\boxed{V_u = 70.92 \text{ KN}}$

a) The maximum shear stress at a distance of 300mm at the supports.

$\tau_v = \frac{3}{2} \frac{V_u}{bd}$

$= \frac{3}{2} \frac{[70.92 \times 10^3]}{250 \times 600}$

$\boxed{\tau_v = 0.70 \text{ N/mm}^2}$

The direct Prestressing force

$f_x = \frac{P}{A} = \frac{1000 \times 10^3}{250 \times 600}$ $(\because f_y = 0)$

$= 6.66$ N/mm^2

$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + \tau_v^2}$

$= \frac{6.66 + 0}{2} + \sqrt{\left(\frac{6.66 - 0}{2}\right)^2 + (0.70)^2}$

$= 6.73$ N/mm^2

$$f_{min} = \left(\frac{f_x+f_y}{2}\right) - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau v^2}$$

$$= \left(\frac{6.66+0}{2}\right) - \sqrt{\left(\frac{6.66-0}{2}\right)^2 + (0.70)^2}$$

$$= -0.79 \ N/mm^2$$

b) If prestressing force required to nullify the shear force at the ~~face~~ phase of the support due to dead and live load.

$V\cancel{\sin\theta} = \theta.$

$V_u - P\sin\theta = 0$

$V_u = P\sin\theta$

$= 1000(0.04)$

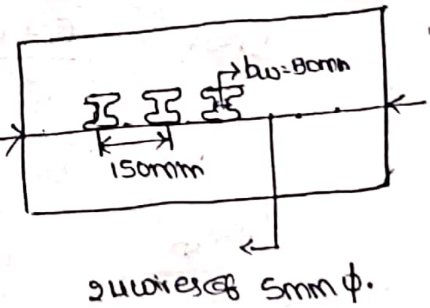
$$\frac{\cancel{76.00} \; 118 \times 10^3}{0.04} = P$$

$\boxed{P = 2950 kN}$

25/2/2020

4. A prestressed I-section has the following properties. Area = 55×10^3 mm^2; second moment of Area (I) = 189×10^7 mm^4; statical moment about centroid = 468×10^4 mm^3 $(A\bar{y})$. Thickness of web = 50mm. It is prestressed horizontally by 24 wires of 5mm diameter. and vertically by by 24 wires at 150mm centre, all the wires carry similar wires a tensile stress of 900 N/mm^2. calculate the principle stresses at the centroid when the shearing force of 80kN acts upon this section.

Sol: Given data;



Area(A) = 55×10^3 mm^2.

Second moment of Area (I) = 189×10^7 mm^4.

$(A\bar{y}) = 468 \times 10^4$ mm^3

Thickness of web = 50 mm.

horizontal = 24 wires of 5mm ϕ

vertical = 24 wires of 5mm ϕ at 150mm centre.

tensile stress = 900 N/mm^2.

shearing force = 80kN.

$$\tau_v = \frac{VA\bar{y}}{Ib_w A\bar{y}}$$

$$= \frac{80\times10^3 (468\times10^{4})}{189\times10^{7} (50) (468\times10^{4})}$$

$$\boxed{\tau_v = 3.96\ N/mm^2}$$

Horizontal Prestress at centre:

$$f_x = \frac{P}{A}$$

P = stress × Area

$= 900 \times 24 \cdot \frac{\pi}{4} (5)^2$

$= 424\ KN.$

$= 424.11\ KN$

$$f_x = \frac{424.11\times10^3}{55\times10^3} = 7.71\ N/mm^2$$

$$\boxed{f_x = 7.71\ N/mm^2}$$

$$f_y = \frac{P}{A} \Rightarrow P = \text{stress} \times \text{Area}$$

$= 900 \times 150 \times 50\ \frac{\pi}{4} (5)^2$

$= 67580\ KN. = 17.67\times10^3\ N.$

$$f_y = \frac{17.67\times10^3}{150\times 50}$$

$$\boxed{f_y = 2.356\ N/mm^2}$$

maximum principal stress:

$$f_{max} = \left(\frac{f_x+f_y}{2}\right) + \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{7.71+2.35}{2}\right) + \sqrt{\left(\frac{7.71-2.35}{2}\right)^2 + (3.96)^2}$$

$$\boxed{f_{max} = 9.81\ N/mm^2}$$

minimum principal stress:

$$f_{min} = \left(\frac{f_x+f_y}{2}\right) - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{7.71+2.35}{2}\right) - \sqrt{\left(\frac{7.71-2.35}{2}\right)^2 + (3.96)^2}$$

$$\boxed{f_{min} = 0.85\ N/mm^2}$$

5) A cantilever Portion of a Prestressed concrete bridge has a rectangular cross-section 600mm wide & 1650mm deep is 8m long. carries a reaction of 350 kN from the suspended span at free end together with a UDL of 60 kN/m inclusive of its own weight. The beam is prestressed by 7 cables each carrying a force of 1000kN of which 3 are located at 150mm, 3 at 400mm & 1 at 750mm from the top edge. calculate the magnitude of the principle stresses at a point 550mm from the top cantilever at the support section.

4/2/2020

Sol: Given data.

c/s section of beam = 600mm × 1650mm.

long(l) = 8m.

Point load = 350 kN at free end.

UDL = 60 kN/m.

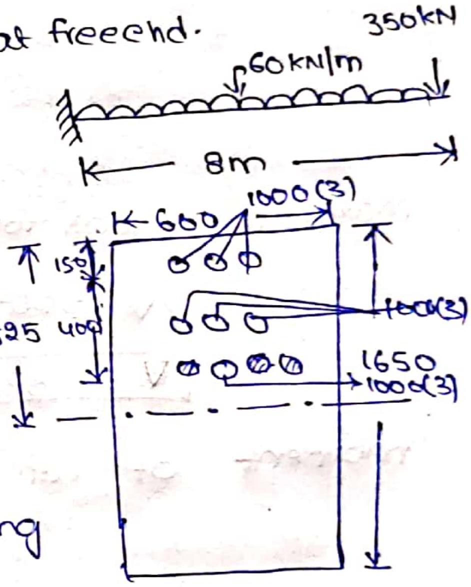
No of cables = 7.

3 at 400mm;

3 at 150mm;

1 at 750mm;

Force (F) = 1000 kN.



4/3/2020

centroid of the prestressing force from the top edge.

$$\bar{y} = A\bar{x}$$

$y_1 = A_1 x_1 = 3000(150)$

$y_2 = A_2 x_2 = 3000(400)$

$y_3 = A_3 x_3 = 1000(750).$

$$\bar{y} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{3000(150) + 3000(400) + 1000(750)}{3000 + 3000 + 1000}$$

$$\boxed{\bar{y} = 342.85mm}$$

Eccentricity (e) = $\frac{d}{2} - \bar{y}$

$= \frac{1650}{2} - 342.85$

825

$\boxed{e = 482.15\,mm}$

Total Prestressing force (P) = 3000+3000+1000

P = 7000 KN.

moment due to prestressing force. = load * 1lar distance

$m = P * e.$

$= 7000 * 482.15$

$m = 3.37 * 10^6$ KN-mm

$\boxed{m = 3.37 * 10^6 \text{ KN-m}}$

$\boxed{m = 3375.05 \text{ KN-m}}$

maximum shear force of cantilever beam

$V = 350 + 60(8)$

$\boxed{V = 830\,kN}$

moment of cantilever beam (m) = moment * distance

$= \cancel{3375.05 * \frac{8}{2}}$

$= 350(8) * \frac{8}{2}$

$\boxed{m = 4720 \text{ KN-m}}$

Moment due to live load + dead load is 4720 KN-m

$f_x = \frac{P}{A} + \frac{Pey}{I} - \frac{my}{I}$ (max. resultant direct stress at 550mm from the top edge of support section).

$I = \frac{bd^3}{12} = \frac{600(1650)^3}{12}$

$\boxed{I = 224.60 * 10^9\,mm^4}$

$f_x = \frac{830}{160}$

$y = 825 - 550$

$= \frac{1650}{2}$

$= 825$

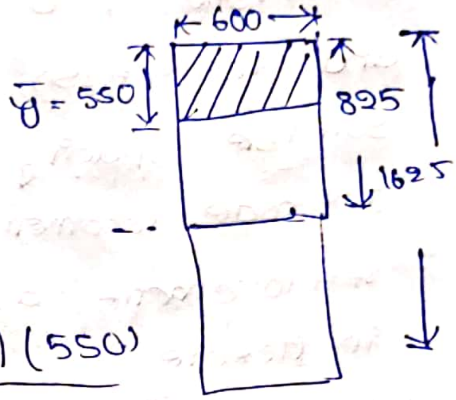
$\boxed{y = 275\,mm}$

$$F_x = \frac{7000*10^3}{600(1650)} + \frac{337*10^6*10^3*275}{224.60*10^9} - \frac{4720*10^6(275)}{224.60*10^9}$$

$$= 7.07 + 4.12 - 5.4$$

$$\boxed{F_x = 5.79 \ N/mm^2}$$

The maximum shear stress of 550mm From the top of the flange.



$$\tau_v = \frac{3}{2} \frac{V A \bar{y}}{Ib}$$

$$A = 550(600)$$

$$A = 330*10^3 \ mm^2$$

$$\tau_v = \frac{(830*10^3)(550*600)(550)}{224.60*10^9*600}$$

$$\boxed{\tau_v = 1.11 \ N/mm^2}$$

$\therefore f_y = 0$

maximum principal stress

$$f_{max} = \left(\frac{f_x+f_y}{2}\right) + \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{5.79+0}{2}\right) + \sqrt{\left(\frac{5.79-0}{2}\right)^2 + (1.11)^2}$$

$f_{max} = 5.99 \ N/mm^2 \simeq \underline{6 \ N/mm^2}$ (compression)

minimum principal stress

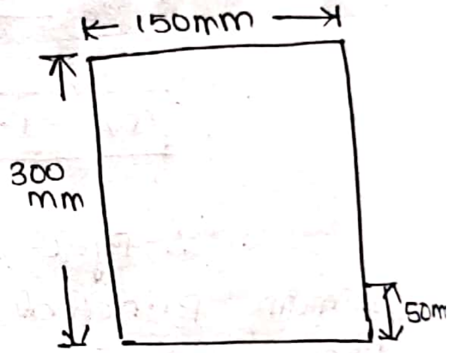
$$f_{min} = \left(\frac{f_x+f_y}{2}\right) - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{5.79+0}{2}\right) - \sqrt{\left(\frac{5.79-0}{2}\right)^2 + (1.11)^2}$$

$$\boxed{f_{min} = -0.205 \ N/mm^2}$$ (Tension)

SHEAR

1. A prestress member of □lar section girder of 150mm wide & 300mm deep to be designed to support and ultimate shear force of 130KN. The uniform prestress across the section is $5N/mm^2$ given the charcteristic cube strength of concrete as $40 N/mm^2$ and Fe415 HYSD bars of 8mm diameter. Design suitable spacing for stirup confirming to the I.S code recomendations. Assume cover to the reinforcement as 50mm & the section uncracked in flexure.



Sol :

$B = 150mm$

$D = 300mm.$

Shearforce (V) = 130KN

ultimate shear

uniform prestress (fp) = $5N/mm^2$

$F_{ck} = 40N/mm^2$

Fe415 HYSD bars of 8mm ϕ

effective cover = 50mm.

uncracked section in flexure:

$V_c = V_{co}$ ∴ Pg No. 32 23.4.1

$V_{co} = 0.67 bD \sqrt{f_t^2 + 0.8 f_{cp} f_t}$ Cl NO: 22.4.1

$f_t = 0.24\sqrt{f_{ck}}.$

$= 0.24\sqrt{40}$

$\boxed{f_t = 1.51 N/mm^2}$

$f_{cp} = 5N/mm^2$

$$V_{co} = 0.67 * 150 * 300 \sqrt{(1.51)^2 + (0.8)(5)(1.51)}$$

$$\boxed{V_{co} = 86.96\ kN}$$

$V > V_{co}$ (or) V_c.

shear force $(130) > 86.96$.

$\therefore$ provide shear reinforcement as per cl NO-23.4.3.2 pg: 33

$$\boxed{\frac{A_{sv}}{S_v} = \frac{V - V_c}{0.87 f_y d_t}}$$

Assume 2legged 8mm ϕ vertical stirrups.

$A_{sv} = 2 \cdot \frac{\pi}{4}(8)^2 = 100.53\ mm^2$.

$$\frac{100.53}{S_v} = \frac{130 - 86.96}{0.87 * 415 * 250}$$

$d_t = d - \text{cover}$.
$= 300 - 50$
$= 250\ mm$.

$$S_v = \frac{100.53 * 0.87 * 415 * 250}{130 * 10^3 - 86.96 * 10^3}$$

$$\boxed{S_v = 210.82\ mm}$$

$S_v \not> 0.75\ d_t$ 22.4.3.2

$= 0.75(250)$

$= 187.5\ mm$

$S_v \not> 4 b_w = 4(150) = 600$

consider less value.

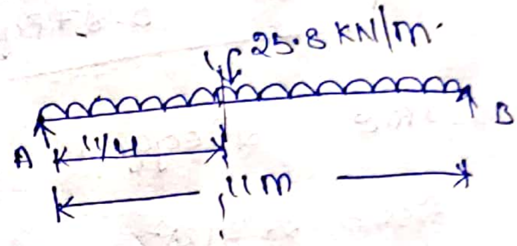
$\therefore$ provide 2legged 8mm ϕ at 187.5 mm c/c.

6/3/2020.

3) A Pretensioned beam of rectangular c/s section 250mm x 550mm has an effective pretensi prestressing force of 900kN at an constant eccentricity of 200mm. It carries a total service load of 25.8 kN/m over an effective span of 11m. Design a shear reinforcement for the beam. The grade of concrete is 40. Design a shear reinforcement at support section & at 1/4th of the span.

Sol:

Cracking flexural:



Given data;

B = 250mm

D = 550mm.

force = 900 kN.

e = 200mm.

load (P) = 25.8 kN/m

span (l) = 11m.

grade of concrete = 40.

At 1/4 span:

$\frac{wl}{2} - wx$.

shear force = load * Distance

RA - wx.

$= \frac{25.8 \times 11}{2} - \left(\frac{11}{4}\right) 25.8$

RA = RB = $\frac{wl}{2}$

$= \frac{25.8 \times 11}{2} - \left(\frac{11}{4}\right) 25.8$

V = 70.95 kN

RA = $\frac{25.8(11)}{2}$

moment (M) = RA·x − w·x·$\frac{x}{2}$ (or) $\frac{3wl^2}{32}$

$= 11 * (12.9) * \frac{11}{4} - (25.8)\ \frac{11}{4} * \left(\frac{\frac{11}{4}}{2}\right)$

= 390225 − 97.56

= 292.66 kN-m

Assume Pre stressing $(f_p) = 1600\ N/mm^2$ in steel.

cracking flexural

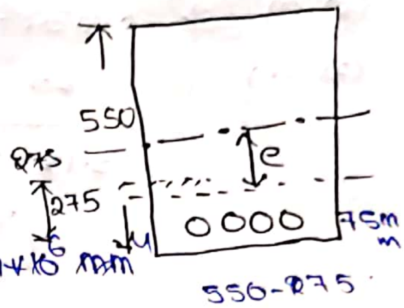
$$V_{cr} = \left(1 - 0.55\,\frac{f_{pe}}{f_p}\right) \tau_o b d + M_o \frac{V}{M}$$

Pg: 47

$f_{pe} = 0.6\, f_p$

$= 0.6(1600)$

$= 960\ N/mm^2$.



$\Rightarrow$ $\boxed{M_o = 0.8\, f_{pt} \cdot \frac{I}{y}}$

$I = \frac{bd^3}{12} = \frac{250(550)^3}{12}$ (500-75) $= 3466.61 \times 10^6\ mm^4$

$I = 2232.74 \times 10^6\ mm^4$

$f_{pt} = \frac{P}{A} + \frac{Pey}{I}$.

$= \frac{900 \times 10^3}{250 \times 550} + \frac{90 \times 10^3 \times 200 \times 200}{2232.74 \times 10^6}$

$y = 275 - 75$
$= 200\, mm$

$f_{pt} = 6.54 + 16.12$

$\boxed{f_{pt} = 22.66\ N/mm^2}$

$M_o = 0.8\, f_{pt} \cdot \frac{I}{y}$.

$= 0.8(22.66)\,\frac{(2232.74 \times 10^6)}{200}$

$\boxed{M_o = 202.37\ kN\text{-}m}$

$\rightarrow$ for τ:

$100 \frac{A_p}{bd} = A_{pe}$ $\therefore$ from table 6 : pg. 47.

$A_p = \frac{P}{f_{pe}}$.

$= \frac{900 \times 10^3}{960}$

$A_p = 937.5\ mm^2$.

$100 \left(\frac{937.5}{250 \times 475}\right) = 0.789$

For M40:

0.75 — 0.6 → (1)

0.789 — τ → (2)

1.00 — 0.68 → (3).

(2)-(1) ⇒ 0.039 — (τ-0.6)

(3)-(1) ⇒ 0.25 — 0.08

$3.12 \times 10^{-3} = 0.25\tau - 0.15$

$$\boxed{\tau_c = 0.61 \ N/mm^2}$$

$$V_{cr} = \left[1 - 0.55 \frac{f_{pe}}{f_p}\right] \tau_c bd + M_0 \cdot \frac{V}{M}.$$

$$= \left[1 - 0.55 \frac{960}{1600}\right] (0.61)(250)(475) + 202.3 \times 10^6 \quad \frac{70.95 \times 10^3}{292.66 \times 10^6}$$

$= 97.58 KN$

9/3/2020

At ~~corner~~ support section:

Shear force (V) = ~~M~~

We have to provide shear reinforcement.

$$\frac{A_{sv}}{b s_v} = \frac{0.4}{0.87 f_y}.$$

Assume A_{sv} = 2 legged 8mm ϕ

$A_{sv} = 2\left(\frac{\pi}{4}\right)(8^2) = 100.53 mm^2$

$$\frac{100.53}{250 (s_v)} = \frac{0.4}{0.87(415)}$$

$$\boxed{s_v = 362.96 mm}$$

At support section:

shear force $(V) = \frac{wl}{2}$.

$= \frac{25.8(11)}{2}$

$V_u = 141.9$ kN.

(pg: 47)

$V_{cr} = 0.1 bd\sqrt{f_{ck}}$

$= (0.1)(250)(475)\sqrt{40}$

$\boxed{V_{cr} = 75.10 \text{ kN}}$

$\boxed{V > V_{cr} \text{ (or) } V_c}$

We have to provide shear reinforcement

22.4.3.2.

$$\frac{A_{sv}}{S_v} = \frac{V - V_c}{0.87 F_y d_t}.$$

Assume Asv is 2legged 8-mm ϕ.

$A_{sv} = 2 \cdot \frac{\pi}{4}(8)^2 = 100.53 \text{ mm}^2$

$$\frac{100.53}{S_v} = \frac{(141.9 * 10^3) - (75.10 * 10^3)}{0.87(415)(475)}$$

$$\frac{100.53}{S_v} = 0.38$$

$\boxed{S_v = 264.55 \text{mm}}$

Check :

$S_v \ngtr 0.75 d_t$

$0.75(475)$

356.25.

$S_v \ngtr 4b_w \Rightarrow 4(250)$

$= 1000.$

Consider less value.

2 legged 8mmφ at 264.55mm. cross-section.